Dark Energy and the CMB

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We find that current Cosmic Microwave Background (CMB) anisotropy data strongly constrain the mean spatial curvature of the Universe to be near zero, or, equivalently, the total energy density to be near critical—as predicted by inflation. This result is robust to editing of data sets, and variation of other cosmological parameters (totaling seven, including a cosmological constant). Other lines of argument indicate that the energy density of non-relativistic matter is much less than critical. Together, these results are evidence, independent of supernovae data, for dark energy in the Universe.

98.70.Vc

Introduction. Cosmologists have long realized that there is more to the Universe than meets the eye. A wide variety of evidence points to the existence of dark matter in the Universe, matter which cannot be seen, but which can be indirectly detected by its contribution to the gravitational field. As observations have improved, the phenomenology of the "dark sector" has become richer. While dark matter was originally posited to explain what would otherwise be excessively attractive gravity, dark energy explains the accelerating expansion—an apparently repulsive gravitational effect. The most well-known argument for this additional dark component is based on inferences of the luminosity distances to high–z supernovae [1]. The anomalously large distances indicate that the Universe was expanding more slowly in the past than it is now; i.e., the expansion rate is accelerating. Acceleration only occurs if the bulk pressure is negative, and this could only be due to a previously undetected component.

Here we argue for dark energy based on another gravitational effect: its influence on the mean spatial curvature. This argument does not rely on the supernovae observations and therefore avoids the systematic uncertainties in the inferred luminosity distances. It is based on a lower limit to the total density, and a smaller upper limit on the density of non-relativistic matter. The lower limit comes from measurements of the anisotropy of the cosmic microwave background (CMB) whose statistical properties depend on the mean spatial curvature [2], which in turn depends on the mean total density. We find that the CMB strongly indicates that $\Omega > 0.4$, where Ω is the ratio of the total mean density to the critical density (that for which the mean curvature would be zero). Upper limits to the density of non-relativistic matter come from a variety of sources which quite firmly indicate $\Omega_m < 0.4$.

The CMB sensitivity to curvature is due to the dependence on curvature of the angular extent of objects of known size, at known redshifts. CMB photons that are penetrating our galaxy today, were emitted from a thin shell at a redshift of $z \simeq 1100$ (called the "last-scattering surface") during the transition from an ionized plasma to a neutral medium. The "object" of known size at known redshift is the sound-horizon of the plasma at the epoch of last-scattering. Its observational signature is the location of a series of peaks in the angular power spectrum of the CMB.

One must be careful about using current CMB data to determine Ω or any other cosmological parameters for several reasons. First, these are very difficult experiments, and the data sets they produce have low signal-to-noise ratios and limited frequency ranges, complicating the detection of systematic errors. Use of different calibration standards further increases the risk of underestimated systematic error. To counter these problems, we examine the robustness of our results to editing of data sets, and check that the distribution of model residuals is consistent with the stated measurement uncertainties.

Second, the CMB angular power spectra depend on a number of parameters other than the curvature. To some degree, a change in curvature can be mimicked by changes in other parameters. We therefore vary six parameters besides the curvature, placing mild prior constraints on some of these so as not to explore unrealistic regions of the parameter space.

Finally, existing data are insufficient to firmly establish the paradigm for structure formation which we have assumed: structure grew via gravitational instability from primordial adiabatic perturbations. Our conclusions *depend* on this assumption. At present, this counts as a possible source of systematic error. Fortunately, future CMB data will verify (or refute) the paradigm and will also allow for the determination of Ω with greatly reduced model dependence [3].

The data. Present data are already so abundant that it must be compressed before it can serve as the basis for a multi-dimensional parameter search. Detector timestreams ("time-ordered data") from a given experiment are reduced to estimates of the signal from the sky, plus a noise covariance. An attempt to do a straight likelihood analysis on the combined maps from all experiments would involve inverting $N \times N$ covariance matrices, with $N \sim 10,000$. While this might be possible once, it is infeasible to do this inversion enough times to systematically explore the large multi-dimensional parameter space. However, the maps and noise covariance matrices can be further reduced to estimates of the angular power spectrum, $C_l \equiv 2\pi \int C(\theta) P_l(\cos\theta) d(\cos\theta)$ where $C(\theta)$ is the correlation function. This final reduction can be viewed as a form of data compression and is called "radical compression" [4] because of the tremendous reduction in the size of the dataset.

Here we use the radically compressed data from `http://www.cita.utoronto.ca/k̃nox/radical.html`. In this compilation the non-Gaussianity of the power spectrum uncertainties has been characterized for a number of experiments with a lot of the weight; assuming Gaussianity leads to biases [4]. The power spectrum estimates, or "bandpowers", are shown in Fig. 1.

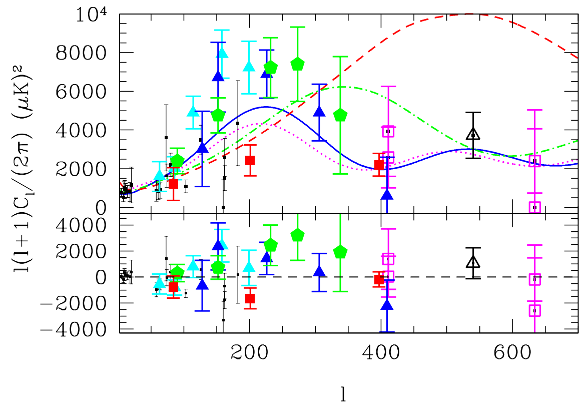

FIG. 1. Constraints on the angular power spectrum: those marked with large symbols are TOCO (filled triangles) [5], CAT (open squares) [6], SK (pentagons) [7], OVRO5M (open triangle) [8] and MSAM (filled squares) [9]. The model curves are standard COBE-normalized CDM (dotted), the best-fit $\Omega = 1$ model (solid), the best-fit $\Omega = 0.4$ model (dot-dashed), and the best-fit $\Omega = 0.2$ model (dashed). The lower panel shows residuals of the best-fit $\Omega = 1$ model.

The Search Method. We search over a seven-dimensional parameter space specified by Ω, $\Omega_b h^2$, $\Omega_{\rm cdm} h^2$, $\Omega_\Lambda h^2$, τ, n_s and C_{10}, where $\Omega_i = \rho_i/\rho_c$ and $i = b, {\rm cdm}, \Lambda$ is for baryons, cold dark matter and a cosmological constant respectively, $\rho_c \equiv 3H_0^2/(8\pi G)$ is the critical density, τ is the optical depth to Thomson scattering, n_s is the power-law index of the primordial matter power spectrum, and C_{10} serves as the normalization parameter. The Hubble constant, $H_0 \equiv 100h\,{\rm km\,sec^{-1}\,Mpc^{-1}}$, is a dependent variable in this space, due to the sum rule: $\Omega_\Lambda + \Omega_b + \Omega_{\rm cdm} = \Omega$. Note that, for specificity and simplicity, we have chosen the dark energy to be a cosmological constant; other choices would lead to negligible changes to our curvature constraints.

For each value of Ω we vary the other parameters and the calibrations on all the experiments to find the minimum value of $\chi^2 = \chi_d^2 + \chi_p^2$. Here χ_d^2 is the offset lognormal form explicitly given in Eq. 39-43 of [4], which was shown to be a good approximation to the log of the likelihood function. Information from non-CMB observations is included as a prior contribution, χ_p^2. Unless otherwise stated, we assume that $h = 0.65 \pm 0.1$ (a reasonable interpretation of several measurements [10]) and $\Omega_b h^2 = 0.019 \pm 0.003$ (from [11] but with a doubling of their uncertainty).

The search over a 24-dimensional space (seven cosmological parameters plus seventeen different calibrations) requires some care. If we write the parameters as some starting set a_p plus corrections δa_p, to minimize the χ^2 one sets $\delta a_p = -\frac{1}{2} F^{-1}_{pp'} \frac{\partial \chi^2}{\partial a_{p'}}$ where $F_{pp'} \equiv \frac{1}{2} \langle \frac{\partial^2 \chi^2}{\partial a_p \partial a_{p'}} \rangle$ is the Fisher matrix for the parameters a_p. If the χ^2 were quadratic (i.e., Gaussian $\mathcal{L}$) then the correction would be exact. Otherwise, χ^2 is approximately quadratic near enough to its minimum, and an iterative application of these equations may converge quite rapidly. A similar procedure is used for estimation of C_ℓ [12].

For the present application, the search may start off far from the minimum and the quadratic approximation is not good enough to allow for rapid convergence. We therefore reduce the amplitude of the correction, to avoid overshooting, as is done in the Levenberg-Marquardt procedure, by increasing the diagonal elements of F until the χ^2 is lowered. We stop the hunt when the new χ^2 is within 0.1 of the old value. We tested this method on simulated data and recovered the correct results.

The Results. Our main results are shown in Fig. 2: the relative likelihood ($\propto \exp(-\chi^2/2)$) of the different values of Ω. Including all the data, the best-fit (minimum χ^2) $\Omega = 1$ model is 2×10^7 times more probable than the best-fit $\Omega = 0.4$ model. $\Omega < 0.7$ is ruled out at the 95% confidence level.

To test the robustness of this result, we edited out single data sets suspected of providing the most weight. Most of these editings produced little change. Only the omission of TOCO changes things substantially, and even then, the best-fit $\Omega = 1$ model is 150 times more probable than the best-fit $\Omega = 0.4$ model. We also edited pairs of data sets: for no CAT and TOCO, no MSAM and CAT, and no MSAM and TOCO, we find $\Omega = 1$ to be 120, 2.5×10^6 and 8 times more likely than $\Omega = 0.4$. Also shown, as measures of goodness-of-fit, are χ^2 and the degrees of freedom. The χ^2 value for the "All" case is a bit high, but one expects even higher ones over 8% of the time, so there is no strong evidence for inconsistencies in the data. As further indication of the robustness of the result, one can see from the "COBE+TOCO" panel of Fig. 2 that it persists even when all but a single *pair* of data sets is removed.

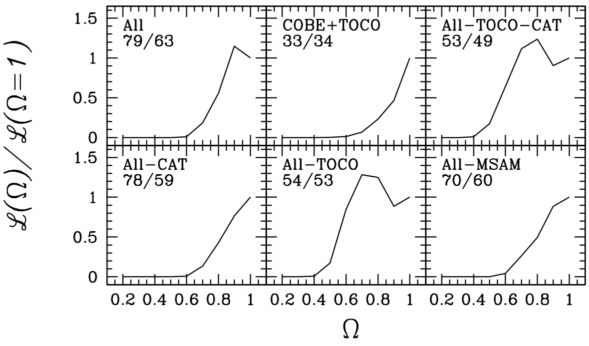

FIG. 2. Relative likelihood of Ω and χ^2 over the degrees of freedom (for $\Omega = 1$) for different collections of data sets.

For the "All" case, the best-fit $\Omega = 1$ model has $\Omega_b h^2 = 0.019$, $h = 0.65$, $\Omega_\Lambda = 0.69$, $\tau = 0.17$ and $n = 1.12$ and is plotted in Fig. 1. There are degeneracies among these parameters though and none of them is strongly constrained on its own. For example, an equivalently good fit (to just the CMB data) is given by the following model with no tilt or reionization: $\Omega = 1$, $\Omega_b h^2 = 0.021$, $h = 0.65$, $\Omega_\Lambda = 0.65$, $\tau = 0$ and $n = 1$.

We also covered the Ω_m, Ω_Λ plane, at each point finding the minimum χ^2 possible with variation of the remaining 5 parameters. Figure 3 shows the $\Delta\chi^2 = 1$, 4 and 9 contours in this plane (where probability drops to 0.6, 0.14 and 0.011 times the probability of the best-fit model, respectively).

Figure 3 also shows constraints on Ω_m from clusters. Although constraints on Ω_m arise from a variety of techniques (for reviews see [13] and references therein) perhaps the most reliable are those based on the determination of the ratio of baryonic matter to dark matter in clusters of galaxies [14–18]. With the assumption that the cluster ratio is the mean ratio (reasonable due to the large size of the clusters) [14,15,17,19], and the baryonic mean density from nucleosynthesis, one can constrain the range of allowable values of Ω_m. Since only the baryonic intracluster *gas* is detected, the upper limits on Ω_m from this method are better understood than the lower limits. Mohr et al. [17] find, from a sample of 27 X-ray clusters, that (including corrections for clumping and depletion of the gas) $\Omega_m < (0.32 \pm 0.03)/\sqrt{h/0.65}$. Including the Hubble constant uncertainty ($h = 0.65 \pm 0.1$) this becomes $\Omega_m < 0.32 \pm 0.05$. Assuming 10% of the baryons to be in galaxies as opposed to the gas, as estimated by [14], we find $\Omega_m = 0.29 \pm 0.05$. Results using SZ measurements are consistent, though less restrictive: $\Omega_m = 0.31 \pm 0.1$ [18]. Most other methods (those that do not rely on the cluster baryon fraction) generally result in formally stronger upper limits to Ω_m. This increases our confidence in the Mohr et al. Ω_m upper limit, but we do not quote these stronger constraints due to our concerns that they are affected by systematic uncertainties that are more difficult to quantify than those in the baryon fraction method.

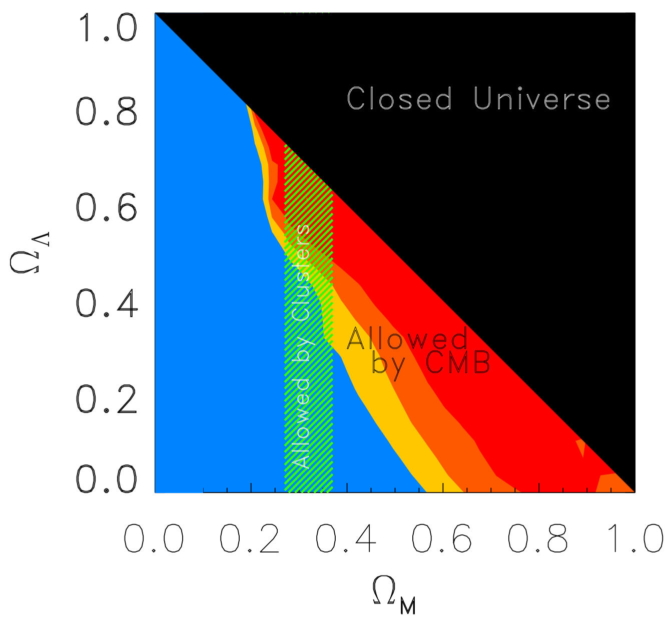

FIG. 3. Likelihood contours in the Ω_m, Ω_Λ plane. Contours show $\Delta\chi^2 = 1, 4, 9$. In green is the 68% confidence region for Ω_m from cluster baryon fraction determinations.

Discussion. There have been a number of other analyses [25] of CMB anisotropy data which generally obtained weaker constraints on Ω [26]. There are technical differences between our work and previous work: we account for the non-Gaussianity of the likelihood function, allow for calibration uncertainties, place "sanity" priors on the Hubble constant and the baryon density, and vary six parameters in addition to the curvature. Also, much of the strength of our argument comes from data reported within the last year. The verdict from the CMB is now in. It does not depend on any one, or even any two, experiments (with the possible exception of COBE). It clearly points towards a flat Universe and, together with cluster data, strongly indicates the existence of dark energy. These conclusions are consistent with, and independent of, the supernovae results. The completely different sets of systematic uncertainties in the two arguments further strengthen the case. Other constraints in the Ω_m, Ω_Λ plane were recently obtained [27] by combining cosmic flow data with supernovae observations.

We have neglected several data sets, all of which, if included, would only strengthen our conclusions. These are PythonV [22], Viper, SuZie [23] and OVRO [24]. PythonV indicates high power, similar to the TOCO and SK data, near $l = 200$. It has not been included because of the strong correlations in the existing reduction of the data; a new reduction with all correlations specified will soon be available. Unpublished Viper data also show a large amount of power near $l = 200$ with less power near $l = 400$. The OVRO and SuZie upper limits in the neighborhoods of $l = 2000$ and $l = 4000$ respectively, favor flat models over open models.

Any model with a broad peak (full-width at half maximum of at least ~ 160) at $l = 320$ or higher will have difficulties agreeing with all the data, due to the combination of the high (relative to COBE) SK and TOCO data points from $l = 150$ to $l = 270$ and the much lower MSAM and TOCO points at $l = 400$ and, less constraining but still significant, the CAT points at $l = 400$. Models fitting this description include the adiabatic models considered here with $\Omega < 0.4$ and also topological defect models, whose breadth is a consequence of the loss of the coherent peak structure [20].

We have been concentrating on implications of the peak location, but the height is also of interest. With fixed h, it is additional evidence for low Ω_m. The lower $\Omega_m h^2$, the later the transition from a radiation-dominated Universe to a matter-dominated Universe and the larger the early ISW effect, which contributes in the region of the first peak [21]. For flat models, the best fit is at $\Omega_m = 0.4$ with $\Omega_m = 1$ four times less likely.

Conclusions. We have shown that $\Omega = 1$ is strongly favored over $\Omega = 0.4$. This result is interesting for two reasons. First, $\Omega = 1$ is a prediction of the simplest models of inflation. Second, together with the constraint $\Omega_m < 0.4$, it is evidence for dark energy.

At present, the CMB says little about the nature of the dark energy. A cosmological constant fits the current data, but then so would many of the other forms of dark energy proposed over the past few years. Generation and exploration of new theoretical ideas as to the nature of this dark energy is clearly warranted.

Measurements of CMB anisotropy have already delivered on their promise to provide new clues towards an improved understanding of cosmological structure formation and fundamental physics. We look forward to greater clarification of the dark energy problem, as well as possibly new surprises, from improved CMB anisotropy measurements in the near future.

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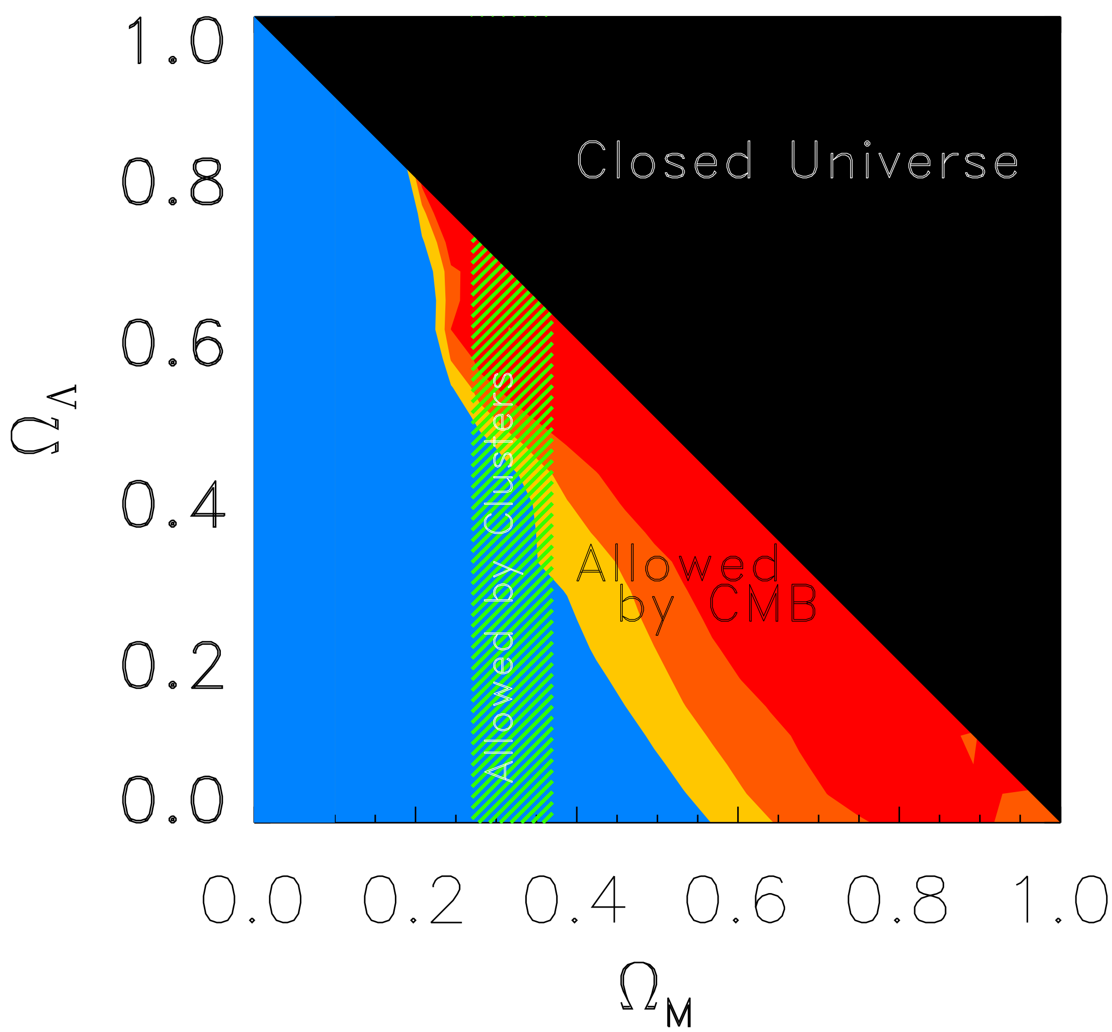

Closed Universe
Allowed by CMB
Allowed by Clusters
1.0
0.8
0.6
0.4
0.2
0.0
Ω_Λ
0.0
0.2
0.4
0.6
0.8
1.0
Ω_M